Mathematical Finance

Introduction to Binary Tree Models, Stochastic Calculus and Black-Scholes Theory

Exercises

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1. Using the data for the single period binary model in Example 1
   (a) compute the replicating portfolio for a put option with strike price $K = \$105$ and exercise time $T = \frac{1}{2}$;
   (b) compute the time 0 put price using the replicating portfolio;
   (c) compute the time 0 put price using the risk neutral probability;
   (d) show how to achieve arbitrage if the time 0 put price were \$5.

2. Prove the *put-call parity* relationship

$$C(0) = P(0) + S(0) - \frac{K}{1+r}$$

   between the prices of a call and put option on the same underlying stock, with the same exercise price K and expiry time T, where $r \geq 0$ is the rate of return on a risk free investment between times 0 and T.

   Hint: If the equality were violated, try to construct an arbitrage portfolio consisting of four securities: call and put options, cash and stock.

3. Consider a five period binary model with the following parameters

$$S(0) = 100, \quad u = 0.04, \quad d = -0.02, \quad r = 0.02.$$

(a) Using a spreadsheet program of your choice compute the price of a European call and a European put option with a strike price $X = 98$ and expiry date $T = 5$.

(b) Using a spreadsheet program of your choice compute the price of an American call and an American put option with a strike price $X = 98$ and expiry date $T = 5$. Determine also at which moment the buyer should exercise these options.

4. Let $C_E(0)$ and $C_A(0)$ denote the prices of a European call and an American call, respectively. Assume that both options have the same strike price and expiry date. Using an arbitrage argument, show that the prices of a European call and an American call option are the same

$$C_E(0) = C_A(0).$$