

Mathematical Finance

Introduction to Binary Tree Models,
Stochastic Calculus and Black-Scholes Theory

Exercises

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AIMS
4–22 January 2010

1. Using the data for the single period binary model in Example 1
 - (a) compute the replicating portfolio for a put option with strike price $K = \$105$ and exercise time $T = \frac{1}{2}$;
 - (b) compute the time 0 put price using the replicating portfolio;
 - (c) compute the time 0 put price using the risk neutral probability;
 - (d) show how to achieve arbitrage if the time 0 put price were \$5.
2. Prove the *put-call parity* relationship

$$C(0) = P(0) + S(0) - \frac{K}{1+r}$$

between the prices of a call and put option on the same underlying stock, with the same exercise price K and expiry time T , where $r \geq 0$ is the rate of return on a risk free investment between times 0 and T .

Hint: If the equality were violated, try to construct an arbitrage portfolio consisting of four securities: call and put options, cash and stock.