Mathematical Finance

Introduction to Binary Tree Models, Stochastic Calculus and Black-Scholes Theory

Assignment 2

Tomasz Zastawniak
Department of Mathematics
University of York
tz506@york.ac.uk

AIMS
4–22 January 2010

Consider the following parameters for the Black-Scholes (BS) model: $r_{BS} = 5\%$, $\mu = 10\%$, $\sigma = 20\%$, $S(0) = 100$ and $T = \frac{3}{12}$ (three months).

The BS model can be approximated by a Cox-Ross-Rubinstein (CRR) model with N time steps of length $\delta = \frac{T}{N}$, and parameters r_{CRR}, u, d such that

$$\begin{aligned} 1 + r_{CRR} &= e^{r_{BS}\delta}, \\ 1 + u &= e^{\mu\delta + \sigma\sqrt{\delta}}, \\ 1 + d &= e^{\mu\delta - \sigma\sqrt{\delta}}. \end{aligned}$$

These formulae are set up in such a way that the return on a cash investment over the time period from 0 to T in the CRR model is the same as in the BS model, and both the expectation and variance of stock price $S(T)$ are the same in the two models.

1. Modify your Python program from Assignment 1 to compute approximate prices of derivatives in the BS model using the CRR model as an approximation. Use this approximation to plot the price $C(0)$ of a European call with maturity T as a function of the strike price K.

2. Plot $C(0)$ as a function of the strike price K using the Black-Scholes formula (soon to come in the lectures)

$$C(0) = S(0)N(d_1) - Ke^{-rT}N(d_2),$$

where

$$\begin{aligned} d_1 &= \frac{\ln \frac{S(0)}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \end{aligned}$$

and where

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy$$

is the standard cumulative normal distribution function.

The graphs from points 1. and 2. should match. Plot them in such a way that they can be compared.