

Real Analysis - Assignment IV

1. If $\{f_n\}$ is a sequence of functions which converge uniformly to the continuous function f on $(-\infty, +\infty)$, prove that

$$\lim_{n \rightarrow \infty} f_n \left(x + \frac{1}{n} \right) = f(x) \quad x \in (-\infty, +\infty)$$

2. Show that the sequence $\{f_n\}$ of functions where

$$f_n(x) = \frac{n}{x+n}$$

is uniformly convergent on $[0, k]$ for each k , but not uniformly convergent on $[0, \infty)$.

3. Show that the sequence $\{f_n\}$ of functions given by

$$f_n(x) = \frac{nx}{1+n^2x^2}, \quad x \in \mathbb{R}$$

does not converge uniformly on \mathbb{R} .

4. Express

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n+k}$$

as a definite integral.