

Real Analysis - Assignment III

Submission deadline: Tuesday (evening), the 15th of December .

1. Suppose f is defined in a neighbourhood of x and suppose $f''(x)$ exists. Show that

$$\lim_{h \rightarrow 0} \frac{f(x+h) + f(x-h) - f(x)}{h^2} = f''(x).$$

Show by an example that the limit may exist even if $f''(x)$ does not.

2. Suppose f is differentiable on $[a, b]$, $f(a) = 0$ and there is a real number A such that $|f'(x)| \leq A|f(x)|$ on $[a, b]$. Prove that $f(x) = 0$ for all x in $[a, b]$.

Hint: fix $x_0 \in (a, b]$, let $M_1 = \sup |f(x)|$, $M_2 = \sup |f'(x)|$ for $a \leq x \leq x_0$. For any such x ,

$$f(x) \leq M_2(x_0 - a) \leq A(x_0 - a)M_1.$$

3. Write the Taylor's formula in powers of x for the function $f(x) = e^x$. Approximate e^x with an error of at most 0,005 in $[-1, 1]$.
4. Determine the accuracy of the approximation

$$\sqrt{x} \sim 1 + \frac{(x-1)}{2} - \frac{(x-1)^2}{8}$$

on the interval $[1/2, 3/2]$.