Real Analysis - Assignment I

Submission deadline: Monday, the 07th of December before 08h30 am.

1. Show that the sequence f where

$$f(n) = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$$

is convergent.

2. Find

$$\lim_{n\to\infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}} \right].$$

3. (a) Find the limit superior and the limit inferior of the sequence:

$$\{(-1)^n(1+\frac{1}{n})\}.$$

(b) Give an example of a bounded sequence $\{a_n\}$ and $\{b_n\}$ such that

$$\limsup(a_n b_n) \neq (\limsup a_n)(\limsup b_n).$$

4. For any sequence $\{s_n\}$, consider the arithmetic means

$$t_n = \frac{s_1 + s_2 + \cdots + s_n}{n}.$$

Prove that $s_n \to s$ implies $t_n \to s$. Give an example of divergent sequence $\{s_n\}$ for which $\{t_n\}$ converges.

5. Suppose $\{x_n\}$ is a sequence of real numbers satisfying $x_{n+1} - x_n = \dfrac{1}{n}$. Does the sequence necessarily converge? Justify or give a counter example.

6. Suppose $\sum a_n$ diverges, $a_n > 0$. Prove that $\sum \frac{a_n}{1+a_n}$ diverges.