Problems: Electrodynamics

1. Maxwell's equations, in the presence of no sources, are

$$\vec{\nabla}\cdot\vec{E} = 0, \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \qquad \vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

In electrostatics we wrote $\vec{E} = -\vec{\nabla}V$; in magnetostatics we wrote $\vec{B} = \vec{\nabla}\times\vec{A}$. Argue that $\vec{E} = -\vec{\nabla}V$ will not, in general, be correct. Argue further that

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}, \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

is perfectly good. What equations do $\vec{A}$ and V satisfy? Show that $\vec{E}$ and $\vec{B}$ are unchanged under the transformation

$$\vec{A} \to \vec{A} + \vec{\nabla}\phi \qquad V \to V - \frac{\partial\phi}{\partial t}$$

We call this a *gauge transformation.*

2. Suppose that a system S' moves relative to a system S with a velocity $\vec{v}$ along the x-axis. When clocks at $S'(x_0', y_0', z_0')$ and $S(x_0, y_0, z_0)$ pass each other they indicate times t_0' and t_0 respectively. Write down the Lorentz transformation formulae for this case.

3. Write down the Lorentz transformation formulae for an arbitrary four vector A^μ without assuming that the velocity $\vec{v}$ of the system S' relative to S is parallel to the x-axis. (Hint: any four vector transforms just like x^μ does; decompose the three vector part of your four vector into a piece parallel to $\vec{v}$ and a piece perpendicular to $\vec{v}$)

4. Derive the velocity addition formulae when the velocity $\vec{v}$ of a system S' relative to S has an arbitrary direction. Give the result in a vector form. (Hint: try using the answer from question 3)

5. Two electron beams travel along the same straight line but in opposite directions with velocities $0.9c$ relative to the laboratory system. Find the relative velocity $\vec{v}$ of the electrons as measured by an observer at rest in the laboratory and also by an observer moving together with one of the electron beams.

When you should hand it in: Write up problem number 5 and hand it in on or before 17:30 on the 26 November.