Quantum mechanics + quantum spin chains

AIMS December 2009 Nepomechie

Introduction [Since everyone is now thinking about the World Cup, it is fun to realize that ..]

Nature is like an honest soccer player

– she likes to play by the rules.

For things like soccer balls, the rules are called "classical mechanics".

For things like electrons and atoms, the rules are called "quantum mechanics".

Quantum mechanics is very important, since it is the foundation for much of physics + chemistry.

The general aim of this course is to introduce you to the rules of quantum mechanics, and to teach you how to play the game – that is, how to apply these rules.

This is a very big subject; and unfortunately, we have only 3 weeks. Obviously, we cannot do it all. This is similar to the problem you have when you arrive in an unfamiliar city — how do you get to know it? I like to choose some specific goal, such as a museum or park, to set a definite direction; and almost always there are more rewards along the way.

We shall do the same thing in this course: our specific goal will be to solve one of the most fundamental problems in quantum mechanics. I will tell you more about it as we get

Just to give you a hint:

farther along! The problem has to do with magnetism. You can think of a (one-dimensional) magnet as a "chain" of many tiny magnets called "spins"

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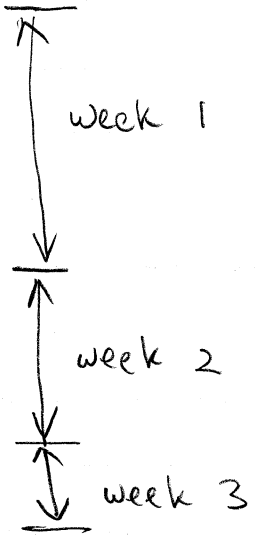
(Hence, the name "spin chain".) These spins interact with each other, and there are many energy levels. Problem: what is the lowest energy? Solution involves "Bethe ansatz"

- very powerful & elegant
- many applications, such as condensed matter physics, statistical mechanics & string theory

This is our "destination". Everything we do will be directed towards formulating or solving this problem. And along the way, you will learn lots of quantum mechanics.

Outline

0. "crash courses" : contour integration
   elementary mechanics
1. classical mechanics [need to formulate QM]
2. linear algebra [main "tool"]
3. principles of quantum mechanics;
   free particle, harmonic oscillator
4. spin
5. Heisenberg spin chain



pre-requisites: elementary calculus (derivatives, integrals)
" linear algebra (vectors, matrices, determinants)

helpful (but we'll review):

- elementary mechanics (Newton's laws, harmonic oscillator)
- " complex variables (contour integration)

disclaimers

- Not traditional intro QM course (no H atom, etc.)
- Not rigorous; few proofs (but many examples!)
- No "philosophy" - emphasis on computation.