Problems involving conditional probability

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1 Written problems

Hopefully we have now covered enough material to consider some rather interesting problems.

1. Box A contains 1 red ball and 3 black balls. Box B contains 1 red ball, 1 white ball and 1 black ball. Box C contains 1 red ball and 1 black ball.
   (a) A box is chosen at random and one ball is chosen. What is the probability that the ball is red?
   (b) Given that the ball chosen is red, what is the probability that the box was A
   (c) Given that the ball chosen is red, what is the probability that the box was B
   (d) Given that the ball chosen is red, what is the probability that the box was C
2. A lie detector test is known to be 80% reliable when the person is guilty and 95% reliable when the person is innocent. Assume we have a suspect from a group of 100 people, only one of whom has ever commited a crime. If the test indicates the person is guilty, what is the probability of innocence?

2 "Fun" problems

These are rather strange, you are not expected to hand them in.

1. Every evening a young man either visits his mother (down town) or his girlfriend (up town). In order to be completely fair, he goes to the bus stop each night at a random time and takes either the uptown bus or the downtown bus according to whichever turns up first. Each bus stops every 15 minutes with perfect regularity. Why does the man only visit his mother twice a month?

2. The exchange problem. You have just won a competion. You are offered one of two envelopes. One envelope contains twice as much money as the other. Having chosen your envelope, the host asks if you wish to change your choice. The host then explains some expected values to you. If the envelope you have chosen contains X, then the other envelope contains either $\frac{X}{2}$ or $2X$. By changing you have a 50% change of receiving $\frac{X}{2}$ and a 50% chance of receiving $2X$. When you calculate the expectation you find that you should change envelopes. But can this be right?

3. The Monty Hall paradox. This is a rather famous problem, which a number of rather famous people have stuggled with. You are asked to consider a game show. The winning contestant gets to pick a prize from one of three doors. One door contains an expensive new car. The other two contain old goats. You do not know what is behind each door. Having selected a door, the host of the show opens one of the two unselected doors, to reveal an old goat. You are offered the chance to switch your selection. Should you alter your selection. Clearly, at the start of this "experiment", $p[Win] = \frac{1}{3}$, but what is it after a door has been opened. Is it now $\frac{1}{2}$? Is it something else? Are you better off staying with your original choice, or switching?