Tutorial-2

Date: March 9th, 2010

1. We want to define the integral

$$\mathcal{I}(\gamma,\omega) = \int_0^\infty dz\ e^{(1/\hbar)f(z)} = \int_0^\infty dz\ e^{(1/\hbar)(\gamma z^4 - \omega^2 z^2)} \tag{1}$$

   (a) For which values of γ does the integral converge?

   (b) We want to rotate the contour of integration by an angle θ in the first quadrant. Does the integrand have any pôles or singularites?

   (c) If the contour of integration goes from the origin to infinity along the ray with fixed polar angle θ, for what values of γ does the integral converge?

   (d) What angle θ do we have to take so that the integral converges for $\gamma > 0$?

2. As $\hbar \to 0$ we can use the method of steepest descent to calculate the integral. We must follow the path for which the real part of the exponent decreases the most rapidly.

   (a) Our integration contour starts at $z = 0$, which is a critical point of the exponent, $f(z)$. Find all the other critical points of $f(z)$.

   (b) Calculate the real part and the imaginary part of $f(z)$.

   (c) Show that for a path of steepest descent (of the real part of $f(z)$) the imaginary part remains constant.

   (d) Find the path of steepest descent from $z = 0$ to the next critical point on the real axis.

   (e) Show that for an ordinary critical point at z_0, ie. $f'(z_0) = 0$, $f''(z_0) \neq 0$, that the path of steepest descent make a $90°$ turn from the path arriving at z_0. In our specific case, the path turns at $90°$ and heads into the first quadrant.