Tutorial

Date: February 26th, 2010

1. Consider the periodic potential

$$V(z) = A(1 - \cos \omega z)) \tag{1}$$

and the action

$$S_M = \int dt \left(\frac{1}{2} \dot{z}(t)^2 - V(z) \right). \tag{2}$$

   (a) What are the classical minimum energy configurations in this theory and what is their degeneracy?

   (b) We want to calculate the effects of tunnelling in the corresponding quantum theory. If A is large, tunnelling is negligible. What are then the approximate low energy states of the quantum theory? (We approximate each well with an appropriate harmonic oscillator well.)

   (c) What is the Euclidean action for this theory and what is the Euclidean equation of motion?

2. Calculate the instanton solution which interpolates from a given well to the well immediately to its right ($z = 2n\pi \to z = 2(n+1)\pi$). Calculate the corresponding action S_0.

3. Calculate the matrix element

$$\langle z = 0 | e^{-\frac{\beta}{\hbar} \hat{h}(\hat{X}, \hat{P})} | z = 2n\pi/\omega \rangle \tag{3}$$

in the semi-classical limit $\hbar \to 0$ and for large $\beta \to \infty$. Find an expression for the matrix element as a sum over multi-instanton configurations and all of the ensuing factors.

4. Use the identity

$$\delta_{n,m} = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i\theta(n-m)} \tag{4}$$

and find the spectrum and structure of the low-lying states of the quantum theory.