Sketching [Français: *esquisser*]

Not to be handed in or graded. In a class next week, we will go through many of these sketches, probably starting with S.1–9. It would be good if you have looked at S.1–9 by Monday.

S.1 Sketch $y_1(x) = x$ and $y_2(x) = x - x^2$ on a single diagram.

S.2 Sketch $E(r) = \dfrac{2}{r^2} - \dfrac{10}{r}$ for positive r.

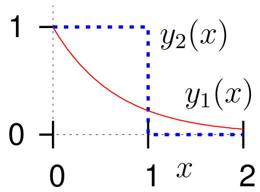
S.3 Sketch $x - 1$, $\log_e(x)$, $\log_e(x^2)$, and $\log_e(3x)$ on a single diagram.

S.4 Sketch $10x$ and e^x on a single diagram.

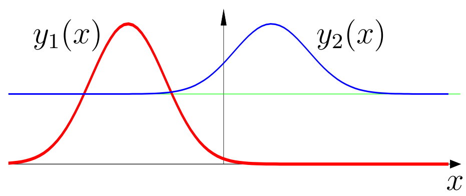
S.5 Sketch $x(1 + 2x)/(1 - x)$.

S.6 For each sketch of $y_1(x)$ and $y_2(x)$ below, draw a sketch of $\int_0^x y_1(x)\mathrm{d}x$ and $\int_0^x y_2(x)\mathrm{d}x$, on a single diagram.

(a)

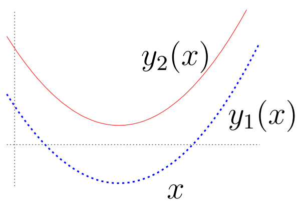


(b)

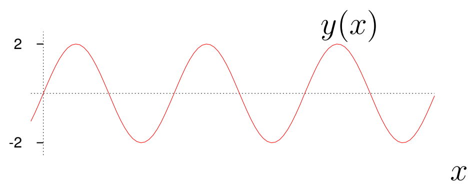


S.7 (a) For $y_1(x)$ and $y_2(x)$ below, sketch of $(y_1(x))^2$ and $(y_2(x))^2$, on a single diagram.
(b) Sketch $(y(x))^2$.

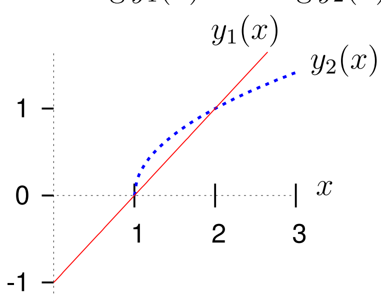
(a)



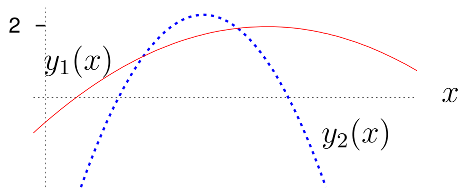
(b)



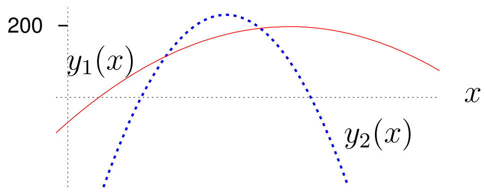
S.8 For $y_1(x)$ and $y_2(x)$ below, sketch $\log y_1(x)$ and $\log y_2(x)$, on a single diagram.



S.9 For each $y_1(x)$ and $y_2(x)$ below, sketch $\exp y_1(x)$ and $\exp y_2(x)$, on a single diagram. (Note carefully the scales on the vertical axes.)



(a)



(b)

S.10 On a single diagram, sketch a Gaussian probability density with mean 2 and standard deviation 2, and a Gaussian with mean 1 and standard deviation 1. Indicate the approximate probability density at the mean of each density.

The definition of a Gaussian density with mean μ and standard deviation σ is

$$P(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

S.11 Sketch x^3 and e^x. Sketch e^x and $\log_e x$. Sketch $\log_e(1+x)$ and x and $\log_e(1-x)$.

S.12 Sketch $\log_e[x^3(1-x)^6]$ and $x^3(1-x)^6$ on two diagrams, one above the other.

S.13 Sketch $1+x^2$ and $1/(1+x^2)$ on a single diagram.

S.14 Sketch $1-x^2$ and $1/(1-x^2)$ on a single diagram.

S.15 Sketch $y(x) = x(x+|x|)$. [$|x|$ denotes the absolute value of x.]

S.16 Sketch $\cos(x)$ and $1/\cos(x)$ for $x \in [-\pi/2, \pi/2]$ on a single diagram.

S.17 Find a function $f(x)$ with maximum at $x = 7$ and with $f(4) = 5$.

S.18 Suppose that we have a bent (unfair) coin which when tossed has a probability of landing heads up that is different from $\frac{1}{2}$. In particular, consider a coin for which $P(\text{heads}) = 0.1$. Let the total number of tosses be N, and the total number of heads obtained be r.

(a) For $N = 2$ tosses, what is the probability distribution of r? [*Hint for checking your answer*: What is the probability of *hh*, *ht*, *th*, *tt*? (h = heads, t = tails)]

(b) Now the coin is tossed N=1000 times. *Sketch* the probability distribution of r. Mark on your sketch the *mean* number of heads, and the *standard deviation* of the number of heads.

['The *mean* of r' has identical meaning to 'the *average* of r', and 'the expected value of r'. Common notations for the mean of r include $\bar{r}$, $\mathcal{E}[r]$, and $\langle r \rangle$.]

[For background reading, see pages 100, 143, and 146 in S. E. Hodge, M. L. Seed *Statistics and probability* (QA273.H655X). See also Alan's exercises; you could use the convolution function `conv` on the computer to help you work out your answer.]

S.19 Sketch $xe^{-x/4}$. Sketch $x^2e^{-x/4}$. Sketch $\frac{1}{x}e^{-10/x}$.

S.20 Sketch $x \log_e x$ and $x \log_2 x$.

S.21 Sketch $H_2(x) = x \log_2 \frac{1}{x} + (1-x)\log_2 \frac{1}{1-x}$.