

Probability and inference practice exercises

(*not* to be handed in or graded)

If you have difficulties, please consult one of these books in the library.

- QA273.H655X S. E. Hodge, M. L. Seed *Statistics and probability*. (Excellent A-level book, highly recommended.)
- QA39.B1X Bostock, L. *Core maths for A-level*. (Gives a good review of basic probability.)
- QA276.C1X Crawshaw, J. *A Concise course in A-level statistics with worked examples*. (P. 134–259 highly recommended – lots of worked solutions!)
- QA401.R54X Riley, K. F., 1936- *Mathematical methods for physics and engineering*. Pages 961–978. (Rapid review, with a few examples.)
- QA39.2 Bostock, L. *Further mechanics and probability*.
- QA273.T48 Tijms, H. C. *Understanding probability: chance rules in everyday life*. (Lots of interesting examples.)

A.1 A club has 50 members. In how many ways can a president, vice president, secretary, and treasurer be chosen?

If the club decides instead to choose a committee of 4 members, who will share all their tasks, in how many ways can the committee of 4 be chosen?

A.2 In how many ways can the letters of the word STATISTICS be arranged?

[Solution: Crawshaw p. 179.]

A.3 A pack of cards contains 52 cards. 13 of the cards are hearts (\heartsuit), 13 are spades (\spadesuit), 13 are diamonds (\diamondsuit), and 13 are clubs (\clubsuit). The 13 hearts are labelled $A\heartsuit, 1\heartsuit, 2\heartsuit, 3\heartsuit, \dots, 10\heartsuit, J\heartsuit, Q\heartsuit, K\heartsuit$. We call the card labelled $A\heartsuit$ ‘the ace of hearts’. The spades, clubs, and diamonds are labelled similarly. So, there are four aces in a pack of 52 cards, and 13 hearts; one card is both an ace and a heart, namely the ace of hearts.

Calculate the probability of drawing **an ace or a heart** from a pack of cards.

[Solution: Riley p. 968.]

A.4 Ten pupils are placed at random in a line. What is the probability that the two youngest pupils are separated?

[Solution: Crawshaw p. 180.]

A.5 In the card game of bridge, each of four players is dealt 13 cards from a full pack of 52 cards. What is the probability that each player receives an ace?

[Solution: Riley p. 978.]

A.6 Four balls are thrown independently at random into four bins. Each ball has an equal probability, $1/4$, of going into each bin.

- What is the probability that, at the end, every bin contains exactly one ball?
- Assuming that every bin does contain one ball, and that the bins are coloured red, black, white, and green, and that the balls are coloured red, black, white, and green, what is the probability that every ball is in a bin of its own colour?

A.7 Find the probability of obtaining at least one 6 when 5 dice are thrown.

[Solution: Crawshaw, p. 175.]

A.8 One card is white on both faces; one is black on both faces; and one is white on one side and black on the other. The three cards are shuffled and their orientations randomised. One card is drawn and placed on the table.

- (a) What is the probability that its upper face is black?
- (b) It turns out to be the case that the upper face of the card *is* black. Now, what is the probability that the *lower* face is black?

If you are not sure about your answer, do an experiment – try sixty repetitions.

A.9 A random variable x has probability distribution

$$P(x = 1) = \frac{1}{3}, P(x = 2) = \frac{1}{4}, P(x = 4) = \frac{1}{5}, P(x = 8) = \frac{1}{6}, P(x = 16) = \frac{1}{20}. \quad (1)$$

What are (a) the median; (b) the mode (most probable value); (c) the mean (also known as the *expected value* or *average* of x) of this distribution.

If $T = 18$ independent outcomes x_1, x_2, \dots, x_{18} are created at random from this distribution, and if c_1 is the count of the number of times that the outcome $x = 1$ occurs, what is the mean of c_1 ? What is its variance, and standard deviation?

A.10 A drunkard takes random steps along a straight road. Each second, he steps either one metre to the right or one metre to the left. The probabilities of right and left are equal. Let his location at time $t = 0$ be $x(0) = 0$ metres. At time $t > 0$, his location will be $x(t)$ metres. What are the mean and variance of $x(t)$?

Optional extra: write a simulation to confirm your theoretical answer.

A.11 A bag contains four balls: two red, and two black. Fred draws a ball from the bag (and doesn't put it back), then George draws a ball. (a) What is the probability that George's ball is red? (b) What is the probability that George's ball is red, *given that* Fred's ball is red? (c) What is the probability that Fred's ball is red, given that George's is red?