

AIMS Problem sheet 1

This is not an exam; you will not be assessed! We will give you the solutions to mark it yourselves.

Don't worry if you find some questions unfamiliar or difficult. Skip those questions and be happy! The purpose of these problems is to help us decide how to teach each other, and how best to use our time at AIMS.

DIRECTIONS

- **Work by yourselves.**
- Don't use calculators, computers, or books.
- Show your work in the spaces provided, and write clearly! Don't worry about presentation or style.
- **Don't write your name on the paper** (we are not going to mark your work).
- If you do not understand the meaning of any question, please ask any of us.
- **Don't spend more than five minutes on each question.** If you are stuck on a question or a part of a question, **don't worry**: Skip the question or part, put a line through it so we know you're skipping it, and return to it later if you have time.

AT 5PM ON TUESDAY

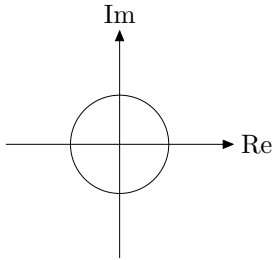
- Bring your work to the classroom **Tuesday at 5pm.**
- We will give you solutions to the exercises and will ask you to mark each others' work, using a red pen.
- When your work has been marked, please hand it in, so we can decide how to improve our teaching.
- **Make a note of the random, unique 5-digit paper number** that is at the head of each page. Using it you can retrieve your own paper later, yet you can do the problems anonymously.

ON WEDNESDAY

- We will hand your work back to you. Make that sure that you understand everything about each question.
- **For any question that gave you any trouble**, come to ask one of us and **we'll explain it.** To the left of each question number is the name of whom to ask. [To divide the work among us.] Alternatively, **ask another student.** We want you all to be teachers!

Anthony **1** *Complex numbers*

For $z = 1 + i$, draw z and z^4 on the complex plane below (with a unit circle drawn to show the scale):



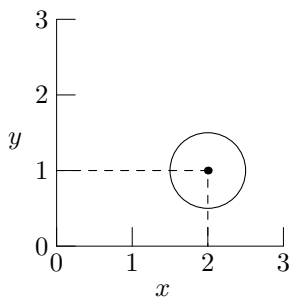
Check your drawing by expanding the binomial to find z^4 .

David **2** *Absolute*

Sketch $y(x) = x + |x|$ for x from -10 to 10 . [$|x|$ denotes the absolute value of x .]

Tendai **3** *Circle*

What is the equation of the following circle?

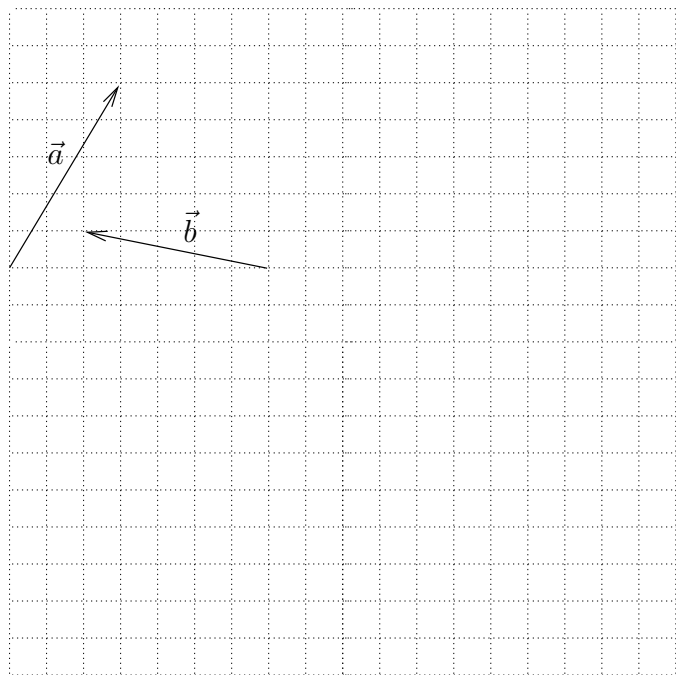


Archie **4** *Estimation*

Estimate (to within a factor of 2) how fast you walk. Remember to include units!

Mike **5** *Combining Vectors*

Below are two vectors \vec{a} and \vec{b} . Draw the sum $\vec{a} + \vec{b}$, and the difference $\vec{a} - \vec{b}$.

*Carl* **6** *Logarithms*

Approximate $\log 72$ given that $\log 2 \approx 0.7$ and $\log 3 \approx 1.1$.

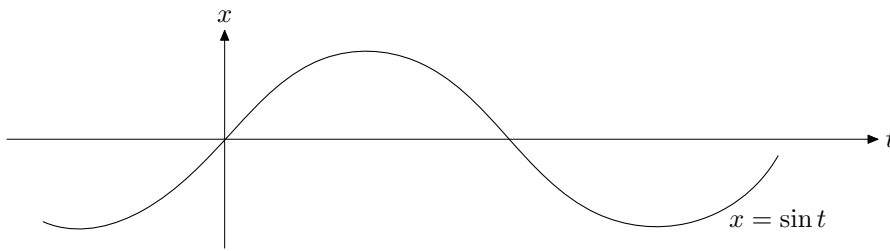
Anthony **7** *Orbits and Satellites*

Around the Earth, there are many satellites in orbit. Which force or forces attract them towards the Earth?

(a) Magnetism; (b) Solar force; (c) Gravity; (d) Friction; (e) Centrifugal force; (f) There is no force.

David 8 Sine

Here is a graph of $x = \sin t$ (solid curve, with t in radians):



Sketch $x = \sin(t-1)$ on the graph.

[Hint: remember that $\sin \pi = 0$, and $\pi \simeq 3$.]

Tendai 9 Melrose

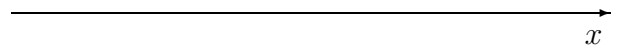
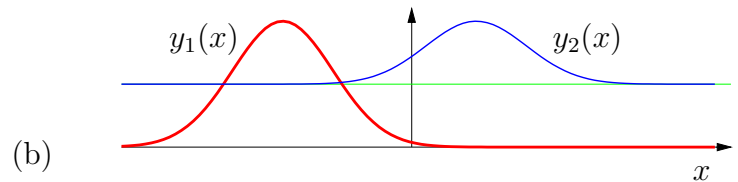
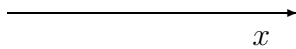
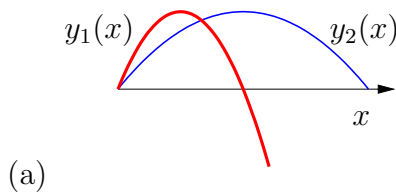
Melrose Road is 100 m long. At $t = 0$ s, you start walking from one end of Melrose Road ($x = 0$) at constant speed 2 m s^{-1} . At $t = 20$ s, Fritz starts walking from the other end ($x = 100$ m) towards you with constant speed 1 m s^{-1} . Sketch x_{you} and x_{Fritz} on the graph:



Set up and solve the equations needed to find *where* on Melrose Road you pass Fritz and *when* that happens. Label the location and time on the graph.

Archie 10 Sketching derivatives

For each sketch of $y_1(x)$ and $y_2(x)$ below, draw a sketch of $\frac{dy_1}{dx}(x)$ and $\frac{dy_2}{dx}(x)$, on a single diagram.



Mike 11 Die

Suppose you have a *fair* six-sided die. (*Fair* means that when the die is thrown it is equally probable that any of the six sides will land face up.) Now suppose that you throw the die three times. What is the probability that you will get:

- a 1 on the first throw, a 2 on the second throw, and a 3 on the third throw?
- a 1, a 2, and a 3 in any order?

Carl 12 Golden

Solve for z the equation

$$(z - 1) = \frac{1}{z}.$$

Anthony **13** *Falling*

Georgie drops his glasses from the top of a tall building. The glasses fall one storey in t seconds. How many storeys will they have fallen in $2t$ seconds? (*Ignore air resistance*).

David **14** *Expectation*

Suppose you have a *fair* six-sided die. You roll the die once. What is the **mean value** of the roll? (The value of the roll is the number on the top face, which is a number from 1 to 6). [‘The *mean* of r ’ has identical meaning to ‘the *average* of r ’, and ‘the expected value of r ’. Common notations for the mean of r include \bar{r} , $\mathcal{E}[r]$, and $\langle r \rangle$.]

Now you roll 2 dice. What is the probability that the **sum** of the two values is 10?

What is the **mean** of the sum of the two values?

Tendai **15** *Inverse*

What is the inverse of the matrix

$$\mathbf{M} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}?$$

Archie **16** *Bank*

A bank account gives interest at a rate of $1/3650$ per day. So, for example, if Abel left 1000R in the account on day zero, then on day 1, his account contains

$$1000 + \frac{1}{3650} \times 1000 = \mathbf{1000.365};$$

on day 2, his account contains

$$\mathbf{1000.365} + \frac{1}{3650} \times \mathbf{1000.365} \simeq 1000.55.$$

- (a) Write down a differential equation describing how the amount of money in the account, $x(t)$, varies with time t (in days). [Approximate the discrete system by a continuous one.]
- (b) Solve the differential equation. How much money is in Abel's account after ten years ($t \simeq 3650$ days)?

Mike **17** *Matrix operations*

Find

$$\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}^2.$$

Carl **18** *Binary*

What is the number 85 in binary (base 2)?

Anthony **19** *Powers and logs*

What are 2^4 , 2^5 , 2^6 ? What is 2^{-10} ?

Sketch a graph of 2^x for x from -10 to 6 . Sketch a graph of $\log_2 x$ for x from 0 to 32 .

 2^x $\log_2 x$ *David* **20** *Differentiation*

(a) Differentiate $\cos(x^3)$ with respect to x .

(b) Differentiate y^2e^{2y} with respect to y .

Tendai **21** *Dimensions*

What are the dimensions, in powers of [L], [M], and [T], of: force, energy, and power?
[Hint: write down simple equations involving a force / an energy / a power]

Archie **22** *Eigenvectors*

What are the eigenvalues and eigenvectors of the matrix

$$\mathbf{M} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}?$$