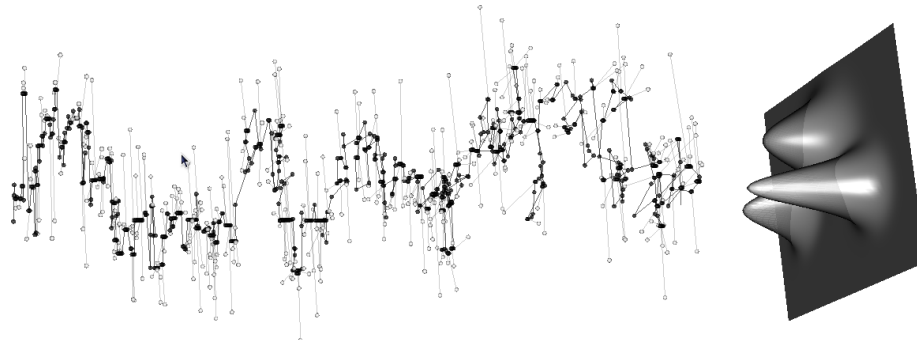


Ioana A. Cosma and Ludger Evers

# Markov Chains and Monte Carlo Methods

Lecture Notes

March 12, 2010



African Institute for Mathematical Sciences





These notes are based on notes written originally by Adam M. Johansen and Ludger Evers for a lecture course on Monte Carlo Methods given at the University of Bristol in 2007.






### Commons Deed

You are free:

-  to copy, distribute and display the work.
-  to make derivative works.

Under the following conditions:

-  **Attribution.** You must give the original authors credit.
-  **Noncommercial.** You may not use this work for commercial purposes.
-  **Share Alike.** If you alter, transform, or build upon this work, you may distribute the resulting work only under a license identical to this one.  
For any reuse or distribution, you must make clear to others the license terms of this work.

Any of these conditions can be waived if you get permission from the copyright holders. Nothing in this license impairs or restricts the authors' moral rights. Your fair dealing and other rights are in no way affected by the above.

This is a human-readable summary of the Legal Code (the full license). It can be read online at the following address:

<http://creativecommons.org/licenses/by-nc-sa/2.5/za/legalcode>

*Note: The Commons Deed is not a license. It is simply a handy reference for understanding the Legal Code (the full license) — it is a human-readable expression of some of its key terms. Think of it as the user-friendly interface to the Legal Code beneath. This Deed itself has no legal value, and its contents do not appear in the actual license.*

---

The L<sup>A</sup>T<sub>E</sub>X source files of this document can be obtained from the authors.

# Table of Contents

<b>Table of Contents</b>	<b>3</b>
<b>1. Markov Chains</b>	<b>5</b>
1.1 Stochastic processes . . . . .	5
1.2 Discrete Markov chains . . . . .	7
1.3 General state space Markov chains . . . . .	18
1.4 Ergodic theorems . . . . .	21
<b>2. An Introduction to Monte Carlo Methods</b>	<b>23</b>
2.1 What are Monte Carlo Methods? . . . . .	23
2.2 Introductory examples . . . . .	23
2.3 A Brief History of Monte Carlo Methods . . . . .	26
2.4 Pseudo-random numbers . . . . .	28
<b>3. Fundamental Concepts: Transformation, Rejection, and Reweighting</b>	<b>31</b>
3.1 Transformation methods . . . . .	31
3.2 Rejection sampling . . . . .	33
3.3 Importance sampling . . . . .	35
<b>4. The Gibbs Sampler</b>	<b>43</b>
4.1 Introduction . . . . .	43
4.2 Algorithm . . . . .	44
4.3 The Hammersley-Clifford Theorem . . . . .	44
4.4 Convergence of the Gibbs sampler . . . . .	46
<b>5. The Metropolis-Hastings Algorithm</b>	<b>53</b>
5.1 Algorithm . . . . .	53
5.2 Convergence results . . . . .	54
5.3 The random walk Metropolis algorithm . . . . .	56
5.4 Choosing the proposal distribution . . . . .	59
<b>6. Diagnosing convergence</b>	<b>63</b>
6.1 Practical considerations . . . . .	63
6.2 Tools for monitoring convergence . . . . .	64
<b>7. State-space models and the Kalman filter algorithm</b>	<b>73</b>
7.1 Motivation . . . . .	73
7.2 State-space models . . . . .	74

7.3	The Kalman filter algorithm . . . . .	76
<b>8.</b>	<b>Sequential Monte Carlo</b>	<b>79</b>
8.1	Importance Sampling revisited . . . . .	79
8.2	Sequential Importance Sampling . . . . .	81
8.3	Sequential Importance Sampling with Resampling . . . . .	84
8.4	Sequential Importance Sampling with Resampling and MCMC moves . . . . .	89
8.5	Smoothing density estimation . . . . .	92
	<b>Bibliography</b>	<b>95</b>