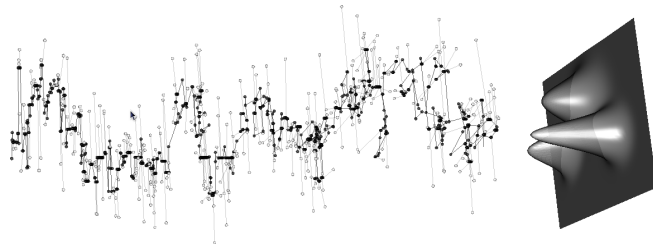


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Markov Chains and Monte Carlo Methods

Lecture Notes




March 12, 2010



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