Sudoku

				5			8	
				6	2			5
6			4			7		
		7				9	6	
		5	2		6	1		
	3	6				4		
		3			7			4
1			5	8				
	6			1				

Sudoku

- the idea of a Sudoku goes back to Leonard Euler: Latin squares
- in our days invented by Howard Garns (USA): Number place
- associate to the places in a Sudoku the variables $x_1, \ldots, x_{81}$ and to each variable x_i the polynomial $F_i(x_i) = \prod_{j=1}^{9}(x_i - j)$
- Let $E = \{(i,j), i < j$ and i, j in the same row, column or $3 \times 3-$box$\}$
- For $(i,j) \in E$ let $G_{i,j} = \frac{F_i - F_j}{x_i - x_j}$.
- Let $I \subset \mathbb{Q}[x_1, \ldots, x_{81}]$ be the ideal generated by the 891 polynomials $\{G_{i,j}\}_{(i,j)\in E}$ and $\{F_i\}_{i=1,\ldots,9}$
- $a = (a_1, \ldots, a_{81}) \in V(I)$ iff $a_i \in \{1, \ldots, 9\}$ and $a_i \neq a_j$ for $(i,j) \in E$

Sudoku

- a well posed Sudoku has a unique solution.
- Let $L \subset \{1, \ldots, 81\}$ be the set of pre-assigned places and $\{a_i\}_{i \in L}$ the corresponding numbers of a concrete Sudoku S.
- Then $I_S = I + < \{x_i - a_i\}_{i \in L} >$ is the ideal associated to the Sudoku S.
- The reduced Gröbner basis of I_S with respect to the lexicographical ordering has the shape $x_1 - a_1, \ldots, x_{81} - a_{81}$ and $(a_1, \ldots, a_{81})$ is the solution of the Sudoku.