AIMS 2011 Gerhard Pfister

Exercises Computer Algebra – Round 3

Deadline for submitting solutions: Thursday, March 10th, 6 p.m..

You must submit solutions to both exercises.

Theoretical Exercise 1. Let $A \in \mathrm{Gl}_m(\mathbb{R})$ and $A \in \mathrm{Gl}_{n-m}(\mathbb{R})$ be matrices defining global monomial orderings $>_A$ resp. $>_B$, and let

$$M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}.$$

Let $>=>_M$ be the global monomial ordering on $K[x_1, \ldots, x_n]$ defined by M. Show that $>$ is an elimination ordering with respect to $x_1, \ldots, x_m$. More precisely, show: If G is a Gröbner basis of an ideal $I \subset K[x_1, \ldots, x_n]$ with respect to $>$, then $G \cap K[x_{m+1}, \ldots, x_n]$ is a Gröbner basis of $I \cap K[x_{m+1}, \ldots, x_n]$ with respect to $>_B$.

Practical Exercise 2. Use the Singular manual to find information on how to solve polynomial equations in Singular. Then write a procedure which computes all **positive** real solutions of a given system of polynomial equations. Apply this procedure to the following system:

```
int n = 8;
ring R= 0,x(1..n),dp;
ideal I =
-1024+x(1)^5*x(3)*x(7),
-1+x(2)^5*x(3)*x(8),
-1+x(4)^5*x(6)*x(7),
-1024+x(5)^5*x(6)*x(8),
x(1)*x(7)+x(2)*x(8)-12*x(7)-x(8),
x(1)+x(4)-13,
x(2)+x(5)-13,
x(7)+x(8)-1;
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