

Clab-Exercises Computer Algebra – Round 2

Practical Exercise 1. Let I be the ideal

$$I = \langle x^2 + 2y^2 - 3, x^2 + xy + y^2 - 3 \rangle \subset \mathbb{Q}[x, y].$$

Use Singular to compute

$$I \cap \mathbb{Q}[x] \quad \text{and} \quad I \cap \mathbb{Q}[y].$$

Practical Exercise 2. Consider the polynomials

$$f_1 = 5ab + bc + 3ad + cd, \quad f_2 = a + b + c + d.$$

Use Singular to check whether

$$f := 5b^2 + 4bc + 8bd + 2cd - 3d^2 \in \mathbb{Q}[f_1, f_2].$$

Practical Exercise 3. Use Singular to compute the kernel of the ring homomorphism

$$\varphi : \mathbb{Q}[x, y, z] \rightarrow \mathbb{Q}[t]/\langle t^{12} \rangle$$

which is induced by sending

$$x \mapsto t^5, \quad y \mapsto t^7 + t^8, \quad z \mapsto t^{11}.$$

Practical Exercise 4. Use Singular to compute the smallest algebraic set B containing the image of the rational parametrization

$$\varphi : \mathbb{A}^2(\mathbb{R}) \setminus V(s \cdot t) \rightarrow \mathbb{A}^3(\mathbb{R}), \quad (s, t) \mapsto \left(\frac{s^2}{t}, \frac{t^2}{s}, s \right).$$