

The Physics of Colliding Balls: Part 2

Not only do we want balls to collide with walls, we would also like balls to collide with one another. We can start by having two balls in a box, and then generalise this to many balls. We will look at the basic mathematics here, but you can add other effects like gravity and friction. Your task is to modify your program of a ball in a box so that the box contains two (or more) balls that collide with each other and with the walls.

Elastic collisions: 1-dimensional

When two balls collide, we need to find the velocities of the balls after the collision, \mathbf{v}'_1 and \mathbf{v}'_2 , in terms of the velocities before the collision, \mathbf{v}_1 and \mathbf{v}_2 . For the 1-dimensional case¹ we do this using conservation of kinetic energy,

$$\frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2, \quad (1)$$

and conservation of momentum,

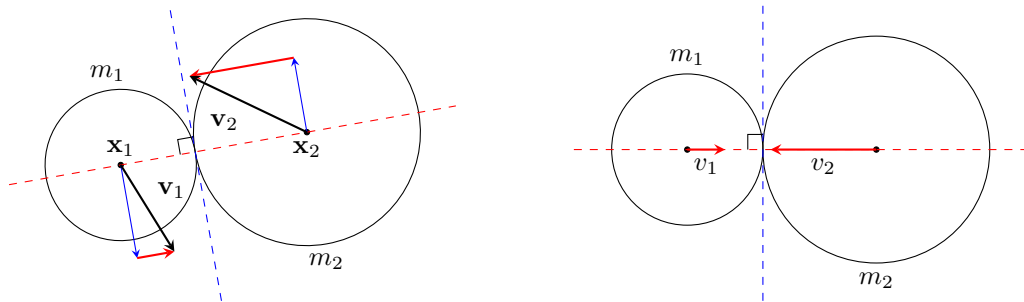
$$m_1v_1' + m_2v_2' = m_1v_1 + m_2v_2, \quad (2)$$

where m_1 and m_2 are the masses of the balls. Solving these equations simultaneously yields

$$v_1' = \frac{v_1(m_1 - m_2) + 2m_2v_2}{m_1 + m_2}, \quad v_2' = \frac{v_2(m_2 - m_1) + 2m_1v_1}{m_1 + m_2}. \quad (3)$$

Elastic collisions: 2- and 3-dimensional

For the higher dimensional cases we can separate the velocities of the balls into two components—one normal (shown in red below) to and one tangent (shown in blue below) to the collision plane.



The tangent component of each velocity will remain unchanged and so we need only update the normal components. By finding the unit normal vector

$$\mathbf{n} = \frac{\mathbf{x}_2 - \mathbf{x}_1}{\|\mathbf{x}_2 - \mathbf{x}_1\|}, \quad (4)$$

where \mathbf{x}_1 and \mathbf{x}_2 are the positions of the centers of the balls, we can find the normal components of the velocities as

$$v_i = \mathbf{v}_i \cdot \mathbf{n} \quad \text{for } i = 1, 2 \quad (5)$$

and then use the 1-dimensional solution (3) to find the normal components of the velocities after the collision.

¹Reference: *Elastic collision* article on Wikipedia