Scientific Programming in Python
Worksheet
2 October 2009

The Lotka-Volterra Model

The Lotka-Volterra model aims to describe the changes in two populations. The first population is known as the prey (let's call them the rabbits, R) and the second as the predators (call them the foxes, F). The differential equations to describe this situation are

$$\frac{\mathrm{d}R}{\mathrm{d}t} = aR - bRF \tag{1}$$

$$\frac{\mathrm{d}F}{\mathrm{d}t} = cRF - dF \tag{2}$$

Here a is the natural birth rate of the rabbits, b is the death rate of rabbits due to being eaten by foxes, c is the growth rate of the foxes dependent on the number of rabbits that they eat, d is the natural death rate of the foxes.

Task 1 Find a numerical solution to this system of equations using the `odeint` function in SciPy. You can try the following values for the parameters to start with—

$$a = 1.5, \quad b = 1.0, \quad c = 0.1, \quad d = 3.0.$$

As for initial populations, you can start with

$$R(0) = 10, \quad F(0) = 5,$$

but be sure to try different initial conditions. You will need to find a good range for your time variable, t. Plot the following graphs of your results.

- R vs t and F vs t on the same graph,
- R vs F (this is know as a phase diagram).

Task 2 In the system of differential equations for the Lotka-Volterra model, the number of rabbits will grow to infinity in the absence of foxes (if $F = 0$). We noted earlier in the course that this is unrealistic and we used the logistic population model instead. What happens if we add a logistic term to (1) to get the ODEs

$$\frac{\mathrm{d}R}{\mathrm{d}t} = aR - bRF - eR^2 \tag{3}$$

$$\frac{\mathrm{d}F}{\mathrm{d}t} = cRF - dF. \tag{4}$$

The new term, $-eR^2$, describes the death rate of rabbits because the environment has a fixed carrying capacity. Try different values for e and describe what effect this has on the plots that you made in Task 1.

Task 3 For each of the two previous tasks, make a vector field plot on the phase space plot. You can use the `quiver` function in Pylab for this. What effect does adding the logistic term (Task 2) have on the phase space?