

Probability in Python

Your task is to implement a Python program for each of the probability problems below. Your program should approximate the probabilities in the problems. The exact solutions have been provided so that you can check your answers. To implement each of these problems, you should ask yourself the following questions:

1. How do I generate the random event(s)?
2. What condition(s) should be true for a positive outcome?
3. How should I count the positive outcomes to approximate the probability?

Useful Python Functions You will need to import the `random` module. This module contains functions for generating random integers, real numbers and permutations. Read the Wiki or use `help('random')` to learn more. The following functions may be useful:

- **choice**: Choose a random element from a list.
- **randint**: Generate a random integer.
- **shuffle**: Order the elements of a list randomly.
- **uniform**: Generate a uniformly distributed real random number.

1. Suppose you have a fair six-sided die. Now suppose that you throw the die three times. What is the probability that you will get:

- (a) a 1 on the first throw, a 2 on the second throw, and a 3 on the third throw? $\left[\frac{1}{6^3}\right]$
- (b) a 1, a 2, and a 3 in any order? $\left[\frac{1}{6^2}\right]$
- (c) three sixes? $\left[\frac{1}{6^3}\right]$

2. Two dice are rolled. We define s as the sum of the values on the two faces.

- (a) What is the mean (expected value) of s ? $[7]$
- (b) What is the probability that s is even? $\left[\frac{1}{2}\right]$

3. Find the probability of obtaining at least one 6 when 5 dice are thrown. $\left[\frac{4651}{7776}\right]$

4. What is the probability of drawing an ace or a heart from a pack of 52 playing cards. $\left[\frac{4}{13}\right]$

5. Box $u = 1$ contains 1 red ball and 3 black balls. Box $u = 2$ contains 1 red ball, 1 white ball, and 1 black ball. Box $u = 3$ contains 1 red ball and 1 black ball.

- (a) A box is chosen at random and one ball is drawn. What is the probability that the ball is red? $\left[\frac{13}{36}\right]$
- (b) Given that the ball is red, what are the probabilities that the chosen box was box $u = 1, 2, 3$? $\left[\frac{3}{13}, \frac{4}{13}, \frac{6}{13}\right]$

6. Three identical skipping ropes are dropped in a heap. The six free ends of the ropes are picked up by three people—each person picks up one free end with his right hand and another free end with his left. Then they all step away from each other.

- (a) What is the probability that every person finds he is holding a single skipping rope? $\left[\frac{1}{15}\right]$
- (b) What is the probability that the three people are joined in a single closed chain? $\left[\frac{8}{15}\right]$