

Scientific Programming in Python

Worksheet 2

22 September 2009

Coin on grid

A coin is placed uniformly at random on a $N \times N$ grid as in Figure 1. We assume that each grid square has unit size and that the coin has diameter d . We saw in Alan's course that the probability that the coin *does not* touch at least one grid line is $(1 - d)^2$.

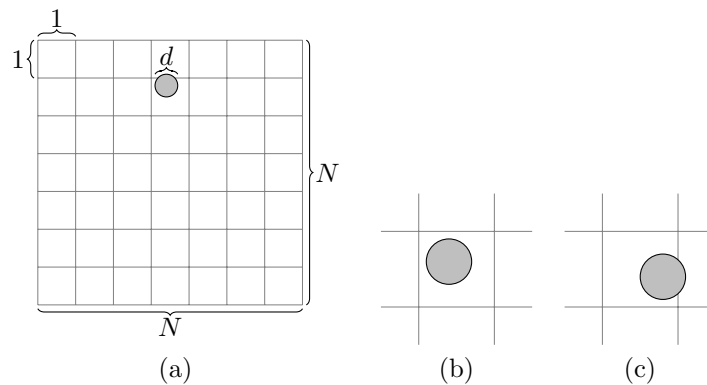


Figure 1: The grid and coin game showing (a) the whole board, (b) a winning position, (c) a losing position.

We are now going to confirm this experimentally by running a large number of random trials. Let's start by writing the code to run the experiment *once*. You should write a program that finds random coordinates for the coin and then determines whether we win or lose the game.

Remember to start your program with

```
from __future__ import division
from scipy import *
```

To draw a uniform random number from the interval $[a, b]$ we can use the function

```
random.uniform(a, b)
```

From which interval should we draw random numbers? Remember that we need a uniform random number for both the x coordinate and the y coordinate of the coin.

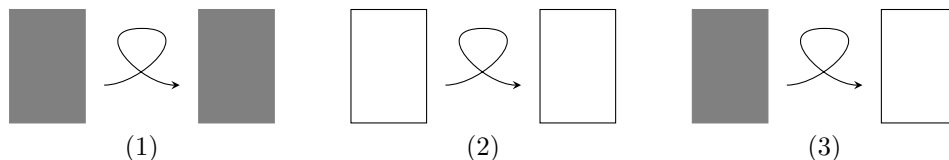
Once we know the x and y coordinates of the center of the coin, we need to determine whether the edge of the coin is touching a grid line. You will need to write down the Python condition for this. This is the hardest part, so take some time to think about it. What should we do if the condition is **True**? What should we do if the condition is **False**?

If you have the code for running the experiment once, use a loop to run it many times. Use variables to record how many times the game is won and how many times it is lost. Check whether

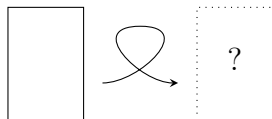
$$\frac{\text{games won}}{\text{total games played}} \rightarrow (1 - d)^2 \quad \text{as} \quad \text{total games played} \rightarrow \infty.$$

Coloured cards

Alan did another probability problem in class where we have 3 cards in a bag. Each of the cards is coloured on both sides. The first card is black on both sides. The second card is white on both sides. The final card is black on one side and white on the other side.



We draw one of the three cards uniformly at random and then look at one of its sides randomly. If we see that this side is white, what is the probability that the other side is also white?



Write a program to run this experiment many times and see what the fraction of successful experiments converges to.

Hints

- You can use the function `random.randint(n)` to generate a random integer from the set $\{0, 1, \dots, n - 1\}$.
- Remember that we are interested only in those experiments where the first side that we observe is white.